Practice 6

Topic: **Research of transient processes of typical dynamic links of the linear ACS**

Purpose: To receive practical skills of a competent research of the transient processes of typical dynamic links on the experimental curve.

Brief data from the theory

Models of linear systems represent analytically in the form of differential equation of the *n* order or system of *n* differential equations of the first order in the form of Cauchy. In practice analytical formulation of the differential equations of ACS elements happens to be quite difficult. At present a lot of ways of an estimation of parameters of ACS elements both in frequency and time fields are developed. It is necessary to note that with an increase of the order of description of ACS elements an accuracy of estimation of parameters decreases sharply.

In the given work the methods of an estimation of parameters of some typical dynamic ACS parts in time field, i.e. on experimentally observed curves of transitional processes will be considered.

It is known transitional process on an output of the part is a reaction to unit step function *l(t)* on an output of the part. Naturally, transitional processes of various types of ACS parts have different forms. Processing the experimental curve of transitional process graphically and analytically, we receive estimators of parameters of a part.

Knowing what type those or other ACS elements belong to and having received estimates of parameters, it is possible to write down their differential equation. Thus, we experimentally receive mathematical description of the investigated ACS elements.

So, a mathematical description of the ACS part by differential equation of the corresponding kind is asked. The task consists in defining of estimators of parameters of the equation.

Inertial link

The differential equation looks like

$$T\frac{dy}{dt} + y(t) = K x(t), \qquad (1)$$

where *x(t)*, *y(t)* – input and output variables respectively; *K*, *T* – required parameters, amplification coefficient and time constant.

Solution of the equation (1) at input action in the form of unit step function *x(t)=1(t)* and zero starting conditions will look like:

$$y(t) = K(1 - e^{-t/T}), \quad y(0) = 0. \qquad (2)$$

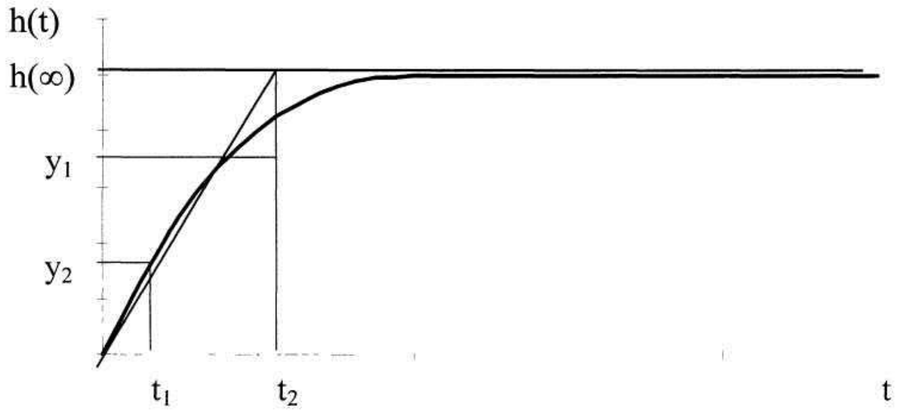

Figure 1 – Transitional process of the inertial part

The way of definition of estimators of parameters is as follows. The amplification coefficient K is defined from the static characteristic, i.e.

$$U_{out} = KU_{in} \ (U_{out} \text{ is taken equal to } h_{steady\text{-}state} = h(\infty))$$

The time constant is defined graphically. For this purpose, at inflection point of the curve of transitional process t_1 we draw a tangent line till crossing with the established value $(h_{steady} = h(\infty))$; then

$$T = t_2 - t_1$$

It is rather convenient to use the following dependence for definition of T:

$$y_2 = 0{,}632(h_{steady\text{-}state} - y_1).$$

Oscillatory link

The differential equation looks like

$$T_2^2 \frac{d^2y}{dt^2} + T_1 \frac{dy}{dt} + y(t) = K x(t), \tag{3}$$

where K, T_1, T_2 – required parameters; $x(t) = 1(t)$.

Roots of the characteristic equation of the vibrational part will be complex, which can be written down in the following way:

$$s_{1,2} = \frac{-T_1}{2T_2^2} \pm j \sqrt{\frac{4T_2^2 - T_1^2}{2T_2^2}} = -\alpha \pm j\beta.$$

Hence.

$$\alpha = \frac{-T_1}{2T_2^2}; \quad \beta = \frac{\sqrt{4T_2^2 - T_1^2}}{2T_2^2}. \tag{4}$$

From (4) we find

$$T_2^2 = \frac{1}{\alpha^2 + \beta^2}; \qquad T_1 = \frac{2\alpha}{\alpha^2 + \beta^2}. \qquad (5)$$

Values of parameters α and β are defined under the diagram of transitional process (figure 2) at zero starting conditions *(y(0) = y(0) =0)*.

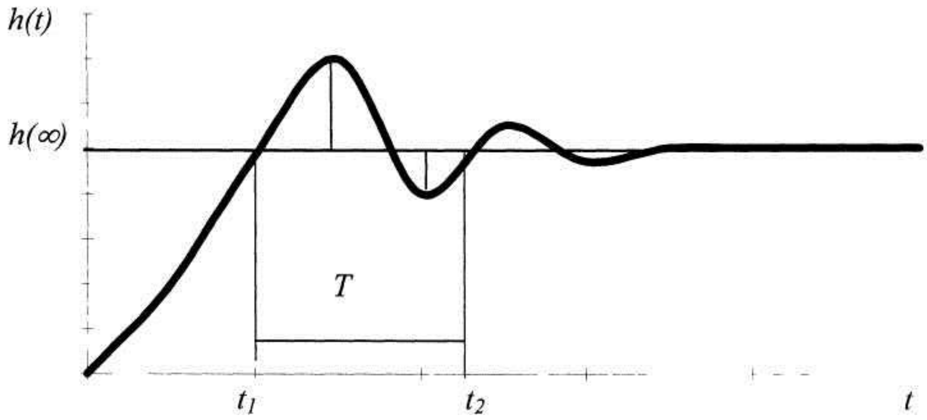

Figure 2 – Transitional process of a vibrational part

For definition of estimators of parameters α and β it is possible to use the following formulas:

$$\beta = \frac{2\pi}{t_2 - t_1} = \frac{2\pi}{T};$$

$$\alpha = \frac{Ln(h2/h1)}{t_2 - t_1} = \frac{Ln(h1/h2)}{t_2 - t_1} = \frac{1}{T} Ln(h_1/h_2) \ ,$$

where h_1, h_2 – values of amplitudes of the first and second fluctuation respectively; T – period of fluctuations.

Having defined α and β, we find estimators of parameters T_1, T_2 under formulas (5).

Solution of the equation (3) at submission to input of *1(t)* unit step function will have the following appearance:

$$y(t) = K\,[1 - e^{-\alpha t}(\cos \beta t + \alpha/\beta \sin \beta t)]. \qquad (6)$$

Aperiodic link of the second order

The differential equation looks like:

$$T_3 T_4 \frac{d^2 y}{dt^2} + (T_3 + T_4)\frac{dy}{dt} + y(t) = K\,x(t), \qquad (7)$$

where $x(t) = 1(t)$; K, T_3, T_4 – required parameters.

Roots of the characteristic equation (7) will be valid (negative) under condition of $T_3 > T_4$ and be equal to

$$s_1 = -\frac{1}{T_3}, \qquad s_2 = -\frac{1}{T_4}.$$

A view of transitional process of the investigated part is presented in figure 3 at zero starting conditions *(at (0) =y (0) = 0)*.

Estimators of parameters T_3 and T_4 are found from the diagram (figure 3),

where $$C = \frac{T_3 T_4}{T_3 - T_4} \ln \; (T_3 / T_4).$$

The way of definition of estimators of parameters is next. At the inflection point of the experimental curve of transitional process we draw a tangent line. Then we define T_4, $(T_3 + T_4)$,graphically, specify length of C.

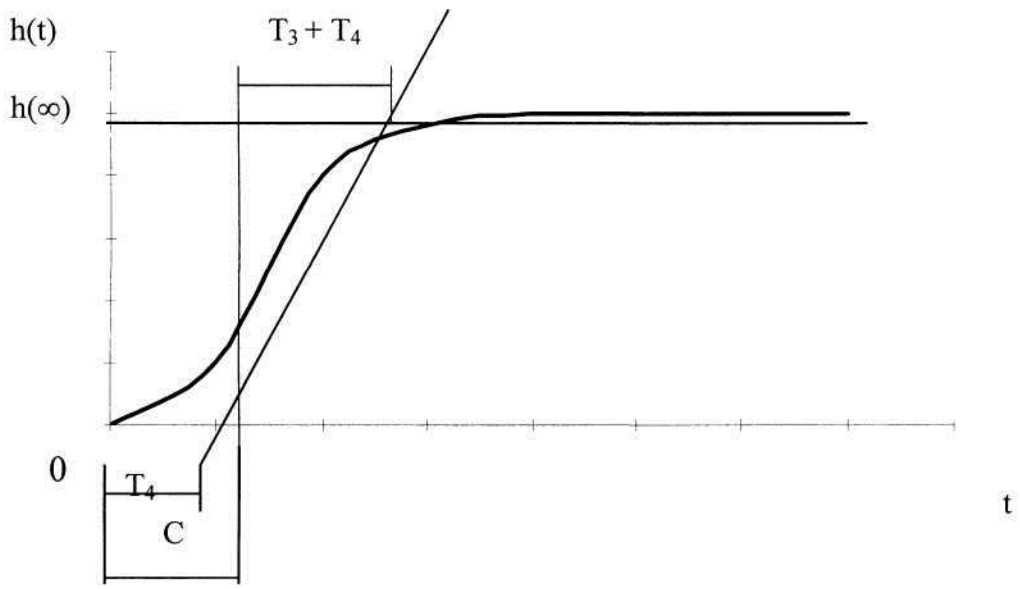

Figure 3 – Transitional process of the aperiodic part of the second order

Solution of the differential equation (7) will have the following view:

$$y(t) = K \left[1 - \frac{T_3}{T_3 - T_4} e^{-t/T_3} + \frac{T_4}{T_3 - T_4} e^{-t/T_4} \right]. \qquad (8)$$

It is necessary to note that this way of definition is suitable at very smooth proceed of the transitional process. Second, in the beginning of coordinates big errors are possible. If initial part of the transitional process adjoins to abscissa axis, the investigated part can be approximated to aperiodic part of the first order with dead time. Transitional process of such part is presented in figure 4.

Transfer function of such part has a following appearance:

$$W(s) = \frac{K}{Ts + 1} e^{-\theta s} \qquad (9)$$

In this case the estimators of parameters are defined by the way described above, and time is count out from θ, i.e. instead of t we put $(t - \theta)$.

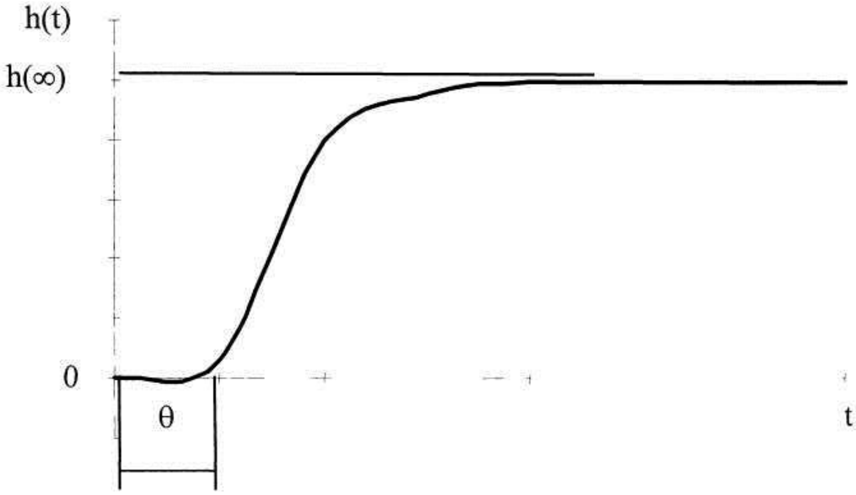

Figure 4 – Transitional process of the part with dead time

Conservative link

The differential equation looks like

$$T^2 \frac{d^2y}{dt^2} + y(t) = K\,x(t), \qquad (10)$$

where $x(t) = 1(t)$; $y(0) = 0$; K, T – required parameters.

A view of transitional process of a conservative part represents continuous fluctuations (figure 5).

The period of the harmonic can be defined in the following way:

$$2\pi \cdot T = t_2 - t_1, \text{ i.e. } T = \frac{t_2 - t_1}{2\pi}.$$

Transitional function at zero starting conditions will have the following

$$h(t) = K\,(1 - \cos \omega_1 t), \quad \omega_1 = 1/T. \qquad (11)$$

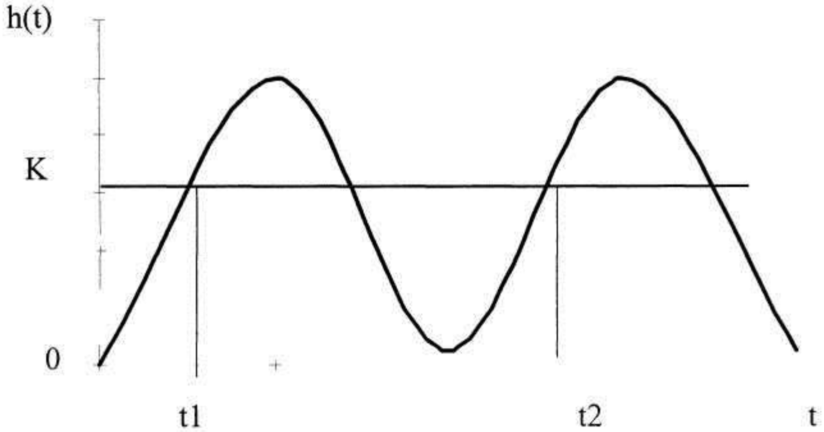

Figure 5 – Transitional process of the conservative part

The Program of researches

1. Take numerical values of parameters of the investigated link from table 2 according to the set by teacher variant.

2. Receive experimental curves of the transitional processes for each part at zero starting conditions, having submitted to input the unit step function *1(t)*.

3. To be convinced of correctness of the received transient processes of typical links of the linear ACS.

INITIAL DATA

Numerical values of parameters of links

Table 2

Variant	1	2	3	4	5	6	7	8	9	10
K	0,4	5,2	0,16	3,8	0,31	7,0	0,84	5,0	0,65	2,7
T	0,22	0,37	0,48	0,61	0,85	1,9	3,5	5,2	7,0	9,2
ξ	0,1	0,2	0,3	0,4	0,5	0,03	0,6	0,08	0,9	0,7
T_1	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
T_2	0,2	0,3	0,4	0,5	0,6	0,5	0,5	0,6	0,6	0,7
T_3	0,4	0,5	0,6	0,5	0,3	0,4	0,5	0,4	0,3	0,5
T_4	0,1	0,2	0,2	0,3	0,1	0,3	0,2	0,1	0,2	0,3

Brief data on the Software

The Software is developed in Windows on Delphi and occupies 1.4 Mb of the operative memory. Operation with Software begins with data entry, "Linear ACS" is carried out in a conversational mode in a natural language in data domain; "Start" is launched by clicking.

Maximum amount of iterations is equal 30. Results are copied into the file result.txt in the current catalogue; the graphic illustration can be seen on the screen of the display.

The Software is developed by the student of Satpayev KazNRU Tokhtabayev A.G.